Digital Circuits

ECS 371

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Lecture 6

Office Hours:
BKD 3601-7 9-10:30 AM
Monday 1:30-3:30
Tuesday 10:30-11:30

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Announcement

- **One more slot for office hours:**
  - **Monday 9:00-10:30**
- I'm not limited to these time slots.
  - Usually in my office (BKD3601-7) from 8AM-5PM

Today

- Some of us participate in the SIIT day activities.
- So, the lecture today will contain no new material.
  - We will do a lot of examples.
- These sides will be posted on the course web site later today.
  - Some of them are the same as what you have as hardcopy

- HW #2 is posted

  Due Next Thursday.

Example

The associative law for addition is normally written as

a. $A + B = B + A$ commutative law for addition

b. $\underbrace{(A + B)} + C = A + \underbrace{(B + C)}$ associative

c. $AB = BA$ commutative law for multiplication

d. $A + AB = A$

Covering

Example

The Boolean equation $AB + AC = A(B+ C)$ illustrates

a. the distribution law

b. the commutative law ✗

c. the associative law ✗

d. DeMorgan's theorem ✗

$$\overline{A+B} = \bar{A}\cdot\bar{B}$$

$$\overline{AB} = \bar{A}+\bar{B}$$

Example

multiplication

The Boolean expression $A \cdot 1$ is equal to

a. A

b. B

c. 0

d. 1

Example

The Boolean expression $A + 1$ is equal to

a. A

b. B

c. 0

d. 1

Example

The Boolean equation $AB + AC = A(B+ C)$ illustrates

a. the distribution law

b. the commutative law

c. the associative law

d. DeMorgan's theorem

Review: Three Useful Rules

These rules do not exist in elementary algebra

(1) $(A+B)(A+C) = A+BC$ ✓

Covering

(2) $A + AB = A$

$A(1+B)$

$= A \cdot 1 = A$

$\square + \square \cdot \triangle$

$= \square$

Examples:

$$XY + XYZ = XY$$

$$W\overline{X}Y + WY = WY$$

$= \boxed{WY} \cdot \overline{X} + \boxed{WY}$

$= WY$

(3) $A + \overline{A}B = A + B$

$(A + AB) + \overline{A}B$

$= A + B(\underbrace{A + \overline{A}}_{1}) = A + B \cdot 1 =$

$A + B$

Examples:

$$\overline{A} + AB = \overline{A} + B$$

$$XY + \overline{XY}Z = XY + Z$$

$$\overline{A} + A\left(\overline{B} + \overline{C}\right) = \overline{A} + \overline{B} + \overline{C}$$

Example

Using Boolean algebra, simplify

$$BD+\underbrace{B(D+E)}_{\cancel{BD}+BE}+\underbrace{\overline{D}(D+F)}_{\cancel{\overline{D}\cdot D}+\overline{D}\cdot F}$$

$x+x=x$

0

$= BD + BE + \overline{D}F$ ← standard form.

$\left(= B(D+E) + \overline{D}F \right)$

Example

$$\overline{X+Y+Z} = \bar{X}\bar{Y}\bar{Z}$$

Using Boolean algebra, simplify

$$\bar{A}\bar{B}C + \underbrace{\overline{(A+B+\bar{C})}}_{=\bar{A}\bar{B}C} + \bar{A}\bar{B}\bar{C}D$$

$$= \underline{\bar{A}\bar{B}C + \cancel{\bar{A}\bar{B}C}} + \bar{A}\bar{B}\bar{C}D$$

$$= \bar{A}\bar{B}(\underbrace{C + \bar{C}D}_{C+D}) = \bar{A}\bar{B}(C+D)$$

$$= \bar{A}\bar{B}C + \bar{A}\bar{B}D$$

Example

Show that

$$\underbrace{(A+B)(A+C)}_{A+\boxed{BC}}(A+D) = A+BCD$$

$$= (A+\square)(A+\Delta) = A+\square\Delta$$

$$= A+BCD$$

$$(A+B)(A+C)(A+D)(A+E) = A+(BCDE)$$

Example

Using Boolean algebra, simplify

$$\left(\underline{B+}\,BC\right)\left(\underline{B+}\,\overline{B}C\right)\left(\underline{B+}\,D\right)$$

$$= B + \left(BC\overline{B}CD\right)$$

$$0$$

$$= B + 0 = B$$

Example

Using Boolean algebra, simplify

$$\underbrace{ABCD + AB\left(\overline{CD}\right)} + \left(\overline{AB}\right)CD$$

$$AB\left(\underbrace{CD + \overline{CD}}_{1}\right)$$

$$= AB + \overline{AB}CD$$

$$= \boxed{AB + CD}$$

$$(X+Y)+Z$$

$$X+(Y+Z)$$

$$X+(Z+Y)$$

$$(X+Z)+Y$$

Example

Using Boolean algebra, simplify

$$ABC\left(AB+\overline{C}\left(BC+AC\right)\right)$$

$\cancel{\bar{C}\cdot C}\cdot(B+A)$

0

$= ABC(AB+0)$

$= ABC(AB) = ABC$

Example

Using Boolean algebra, simplify

$$\underbrace{\overline{A}+A\overline{B}}_{\overline{A}+\overline{B}}+AB\overline{C}$$

$$=\overline{A}+\underbrace{\overline{B}+AB\overline{C}}_{\overline{B}+B(A\overline{C})}$$

$$=\overline{B}+A\overline{C}$$

$$=\overline{A}+\overline{B}+A\overline{C}=\overline{A}+\overline{B}+\overline{C}=\overline{A\cdot B\cdot C}$$

Product Term

A single literal or a product of two or more literals.

Example: $A \cdot \overline{B} \cdot C$

$A \cdot C$

A

$A \cdot \overline{B} \cdot C \cdot D$

$\overline{A} \cdot \overline{B} \cdot \overline{C}$

Caution:

$\overline{A \cdot B \cdot C}$ is not a product term.

Q: When does $A \cdot \overline{B} \cdot C = 1$?

1 0 1

iff

$(A, B, C) = (1, 0, 1)$

$\bar{A}\bar{B}C = 1$

$(ABC) = (0,0,1)$

Example

Find the value of X for all possible values of the variables when

101   011   001   000   100

$$X = A\bar{B}C + \bar{A}BC + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C}$$

Standard sum of products

	A	B	C						
1	0	0	0	1	0	0	0	1	0
1	0	0	1	1	0	0	1	0	0
0	0	1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	0	0	0
1	1	0	0	1	0	0	0	0	1
1	1	0	1	1	1	0	0	0	0
0	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0

Example

Find the value of X for all possible values of the variables when

$A\bar{B}C + A\bar{B}\bar{C} + ABC =$ $X = A\overline{B} + ABC$

10 111

What makes a product term correspond to only 1 row in the truth table?

A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

"C" is free!

A: Each term should have all variables.

Example

Find the value of X for all possible values of the variables when

$$\overset{0\,0}{X = \overline{A}\,\overline{B} + \overline{C} + \overset{1\,1\,1}{ABC}} \leftarrow \text{SOP}$$

A	B	C	X
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

Example

Find the value of X for all possible values of the variables.

A=1, C=0

$$X = \overline{B} + A\overline{C} + A\overline{B}C$$

101

A	B	C	
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

B=0 101 010

$$X = \overline{B} + \overline{C} + A\overline{B}C + \overline{A}B\overline{C}$$

C=0

A	B	C	
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

HW#2 K-map!!