

# Digital Circuits

ECS 371

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Lecture 6

Office Hours:

BKD 3601-7 **9-10:30**

Monday 1:30-3:30 **AM**

Tuesday 10:30-11:30

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# Announcement

- **One more slot for office hours:** ✓
  - **Monday 9:00-10:30** ←
- I'm not limited to these time slots.
  - Usually in my office (BKD3601-7) from 8AM-5PM

# Today

- Some of us participate in the SIIT day activities.
- So, the lecture today will contain no new material.
  - We will do a lot of examples
- These slides will be posted on the course web site later today.
  - Some of them are the same as what you have as hardcopy

• HW #2 is posted

Due Next Thursday.

# Example

The associative law for addition is normally written as

→ a.  $A + B = B + A$  ← commutative law for addition

b.  $(A + B) + C = A + (B + C)$  ← associative

c.  $AB = BA$  ← commutative law for multiplication

d.  $A + AB = A$  ←  
Covering

# Example

The Boolean equation  $AB + AC = A(B + C)$  illustrates

- a. the distribution law
- b. the commutative law ~~X~~
- c. the associative law ~~X~~
- d. DeMorgan's theorem ~~X~~

$$\overline{A + B} = \bar{A} \cdot \bar{B}$$

$$\overline{AB} = \bar{A} + \bar{B}$$

# Example

multiplication



The Boolean expression  $A \cdot 1$  is equal to

a.  $A$

b.  $B$

c.  $0$

d.  $1$

# Example

The Boolean expression  $A + 1$  is equal to

- a.  $A$
- b.  $B$
- c.  $0$
- d.  $1$

# Example

The Boolean equation  $AB + AC = A(B + C)$  illustrates

- a. the distribution law
- b. the commutative law
- c. the associative law
- d. DeMorgan's theorem



# Review: Three Useful Rules

These rules do not exist in elementary algebra

$$\textcircled{1} (A + B)(A + C) = A + BC \quad \checkmark$$

*Covering*

$$\textcircled{2} A + AB = A$$

$$A(1 + B) \\ = A \cdot 1 = A$$

Examples:

$$XY + XYZ = XY$$

$$\rightarrow W\bar{X}Y + WY = WY$$

$$= \boxed{WY} \cdot \cancel{\boxed{X}} + \boxed{WY}$$

$$= WY$$

$$\square + \square \cdot \Delta$$

$$= \square$$

$$\textcircled{3} A + \bar{A}B = A + B \quad \leftarrow$$

$$\underbrace{(A + AB)}_{\textcircled{2}} + \bar{A}B$$

$$= A + B(\underbrace{A + \bar{A}}_1) = A + B \cdot 1 = A + B$$

Examples:

$$\bar{A} + AB = \bar{A} + B$$

$$XY + \bar{X}YZ = XY + Z$$

$$\bar{A} + A(\bar{B} + \bar{C}) = \bar{A} + \bar{B} + \bar{C}$$

# Example

Using Boolean algebra, simplify

$$BD + B(D + E) + \bar{D}(D + F)$$

$$x + x = x$$

$$\cancel{BD} + BE \quad \bar{D} \cdot \cancel{D} + \bar{D} \cdot F$$

$$= BD + BE + \bar{D}F \quad \leftarrow \text{standard form.}$$

$$\left( = B(D + E) + \bar{D}F \right)$$

# Example

$$\overline{X+Y+Z} = \bar{X}\bar{Y}\bar{Z}$$

Using Boolean algebra, simplify

$$\overline{ABC} + \overline{(A+B+\bar{C})} + \bar{A}\bar{B}\bar{C}D$$
$$= \bar{A}\bar{B}C$$

$$= \bar{A}\bar{B}C + \cancel{\bar{A}B\bar{C}} + \bar{A}\bar{B}\bar{C}D$$

$$= \bar{A}\bar{B}(C + \bar{C}D) = \bar{A}\bar{B}(C+D)$$

$$C+D = \bar{A}\bar{B}C + \bar{A}\bar{B}D$$

# Example

Show that

$$\underbrace{(A+B)(A+C)(A+\triangle)}_{A+\boxed{BC}} = A + BCD$$

$$= (A+\square)(A+\Delta) = A + \square\Delta$$

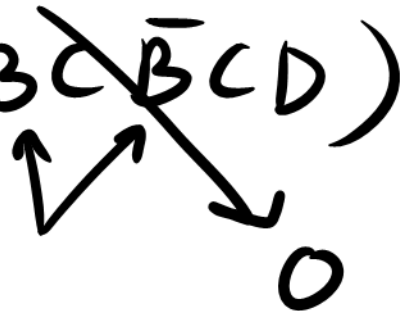
$$= A + BCD$$

$$(A+B)(A+C)(A+D)(A+E) = A + (BCDE)$$

# Example

Using Boolean algebra, simplify

$$(\underline{B} + BC)(\underline{B} + \overline{B}C)(\underline{B} + D)$$

$$= B + (\cancel{BC} \overline{B}C D)$$


$$= B + 0 = B$$

# Example

Using Boolean algebra, simplify

$$ABCD + AB(\overline{CD}) + (\overline{AB})CD$$

$$AB(CD + \overline{CD})$$

1

$$= AB + \overline{AB}CD$$

$$= \boxed{AB + CD}$$

$$(X + Y) + Z$$

$$X + (Y + Z)$$

$$X + (Z + Y)$$

$$(X + Z) + Y$$

# Example

Using Boolean algebra, simplify

$$ABC \left( AB + \overline{C} (BC + AC) \right)$$

$$\cancel{\overline{C} \cdot C} \cdot (B + A)$$

0

$$= ABC (AB + 0)$$

$$= \overbrace{ABC} (AB) = ABC$$

# Example



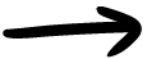


Using Boolean algebra, simplify

$$\begin{aligned} & \overline{A} + \overline{A}\overline{B} + A\overline{B}\overline{C} \\ & \underbrace{\overline{A} + \overline{A}\overline{B}}_{\overline{A} + \overline{B}} + A\overline{B}\overline{C} \\ & = \overline{A} + \overline{B} + \underbrace{A\overline{B}\overline{C}}_{\overline{B} + B(A\overline{C})} \\ & = \overline{A} + \overline{B} + A\overline{C} \\ & = \overline{A} + \overline{B} + A\overline{C} = \overline{A} + \overline{B} + \overline{C} = \overline{A \cdot B \cdot C} \end{aligned}$$




# Product Term

A single literal or a product of two or more literals.

Example:  $A \cdot \bar{B} \cdot C$    
 $A \cdot C$    
 $A$    
 $A \cdot \bar{B} \cdot C \cdot D$    
 $\bar{A} \cdot \bar{B} \cdot \bar{C}$  

Caution:

$A \cdot B \cdot C$  is not a product term.

$101$    
Q: When does  $A \cdot \bar{B} \cdot C = 1$ ?

iff

$$(A, B, C) = (1, 0, 1)$$

# Example

$$\bar{A}\bar{B}C = 1$$

$$(ABC) = (0,0,1)$$

Find the value of  $X$  for all possible values of the variables when

$X = \overline{ABC} + \overline{ABC} + \overline{ABC} + \overline{ABC} + \overline{ABC}$

$\overline{101} \quad \overline{011} \quad \overline{001} \quad \overline{000} \quad \overline{100}$

	A	B	C						
1	0	0	0	1	0	0	0	1	0
1	0	0	1	1	0	0	1	0	0
0	0	1	0	0	0	0	0	0	0
1	0	1	1	1	0	1	0	0	0
1	1	0	0	1	0	0	0	0	1
1	1	0	1	1	1	0	0	0	0
0	1	1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0

Standard sum of products

# Example

Find the value of  $X$  for all possible values of the variables when

$$\underbrace{A\bar{B}C + A\bar{B}\bar{C} + ABC}_{\substack{\downarrow\downarrow\downarrow \quad \downarrow\downarrow\downarrow \quad \downarrow\downarrow\downarrow}} = X = \underbrace{A\bar{B}}_{10} + \underbrace{ABC}_{111}$$

A	B	C
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

0  
0  
0  
0  
1  
1  
0  
1

What makes a product term correspond to only 1 row in the truth table?

"C" is free!

A: Each term should have all variables.

# Example

Find the value of  $X$  for all possible values of the variables when

$$X = \underline{\underline{\overline{00}}} + \overline{C} + \underline{\underline{111}} \quad \leftarrow \text{SOP}$$

A	B	C	X
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

